

Fourier Transforms and its Applications in Engineering Field

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ABSTRACT: Fourier transform provides a continuous frequency resolution of a function. Fourier transform is useful in the study of frequency response of a filter, solution of PDE, discrete Fourier transform and Fast Fourier transform in signal analysis. A Fourier transform when applied to a partial differential equation reduces the number of independent variables by one. The theory of integral transforms afford mathematical devices through which solutions of numerous boundary value problems of engineering can be obtained.

KEYWORDS: Fourier transforms, Fourier sine and cosine Transforms, properties, Inverse Fourier Transforms.

I. INTRODUCTION

This paper deals with a brief overview of what Fourier Transform is and its applications in the industry. The Fourier Transform is a mathematical procedure which transforms a function from the time domain to the frequency domain. Jean Baptiste Joseph Fourier, the French mathematician/physicist made an astonishing discovery in 1800.

According to Fourier, every function could be represented by an infinite series of elementary trigonometric functions sine and cosine. The Fourier Transform is a specific type of integral transform. The Fourier Transform of $f(x)$ is denoted by $F\{f(x)\}$ or $\hat{f}(\alpha)$ or $F(\alpha)$. Also $f(x)$ is called the Inverse Fourier Transform of $F(\alpha)$ or $\hat{f}(\alpha)$. Fourier Transform is useful in the general linear system theory for continuous-time signals and digital signal processing.

Definition of Fourier Transforms : Fourier Transform method is the process of obtaining $F(\alpha)$ for a given function $f(x)$. Fourier Transform breaks up the function into a continuous spectrum of frequencies α .

The Fourier Transform of $f(x)$ is defined by $F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx$

II. PROPERTIES OF FOURIER TRANSFORMS

1.1 Linearity Property :

For any constants a and b , $F\{af(x) + bg(x)\} = aF\{f(x)\} + bF\{g(x)\}$.

1.2 Change of scale Property :

$F\{f(ax)\} = \frac{1}{a} F\left(\frac{\alpha}{a}\right)$, $a > 0$

1.3 Shifting Properties :

i) $F\{f(x-a)\} = e^{-i\alpha a} F(\alpha)$

ii) $F\{e^{i\alpha x} f(x)\} = F(\alpha + a)$

1.4 Modulation properties :

$F\{f(x)\cos ax\} = \frac{1}{2}\{F(\alpha + a) + F(\alpha - a)\}$

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$$F\{f(x)\sin ax\} = \frac{1}{2}\{F(\alpha + a) - F(\alpha - a)\}$$

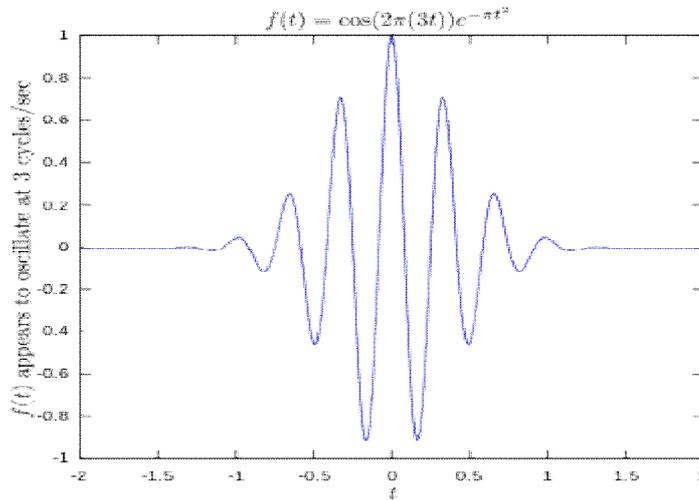
1.5 Transform of the derivative

If $f(x) \rightarrow 0$ as $|x| \rightarrow \infty$ then $F\{f'(x)\} = -iaF\{f(x)\}$.

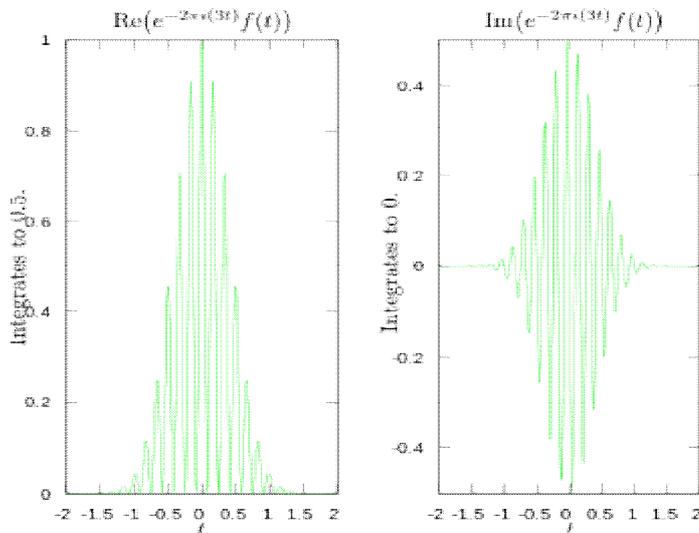
1.6 The derivative of the Transform: If $F(\alpha)$ is the Fourier Transform of $f(x)$, then $\frac{d}{d\alpha}[F(\alpha)] = iF[xf(x)]$

III. GRAPHS OF VARIOUS FREQUENCY OF FOURIER TRANSFORMS

Fourier transform measures how much of an individual frequency is present in a function $f(t)$.



Original function showing oscillation 3 Hz.



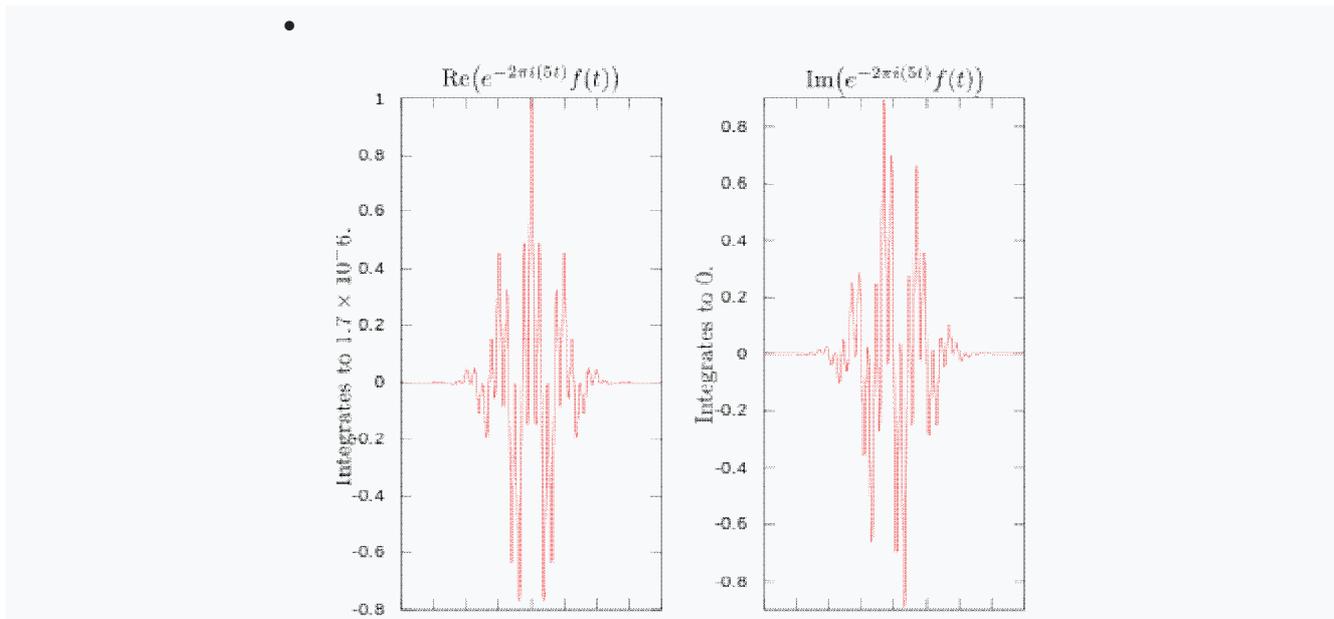
Real and imaginary parts of integrand for Fourier transform at 3 Hz

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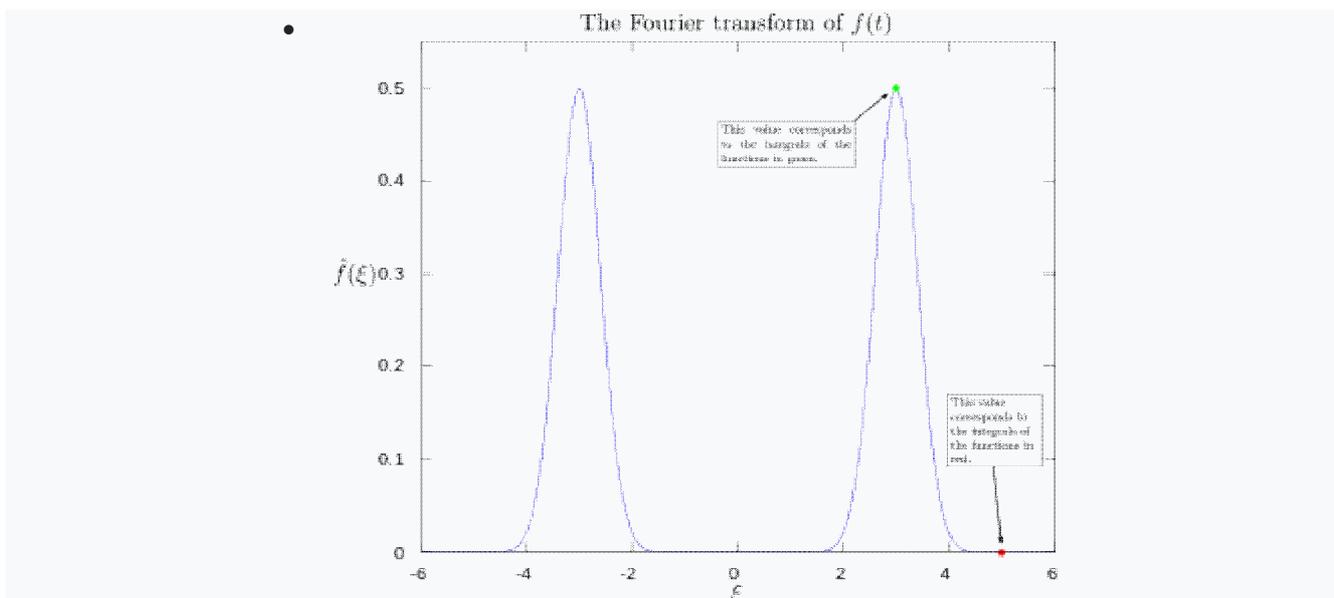
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Real and imaginary parts of integrand for Fourier transform at 5 Hz



Fourier transform with 3 and 5 Hz labeled

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IV. FOURIER SINE AND COSINE TRANSFORMS

Definition of Fourier sine Transform :

For a given $f(x)$, defined for $x>0$, Fourier sine Transform is denoted by $F_s(\alpha)$ and defined as

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin \alpha x \, dx$$

If the Fourier sine Transform $F_s(\alpha)$ of $f(x)$ is known , then $f(x)$ is called the inverse Fourier sine Transform of $F_s(\alpha)$ and defined as $f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_s(\alpha) \sin \alpha x \, d\alpha$.

Relationship between $F(\alpha)$ and $F_s(\alpha)$:

For a function $f(x)$ defined in $(-\infty, \infty)$, the Fourier transform is

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} \, dx = \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{\infty} f(x) \cos \alpha x \, dx + i \int_{-\infty}^{\infty} f(x) \sin \alpha x \, dx \right\} \dots \dots \dots (*)$$

Suppose $f(x)$ is an odd function in $(-\infty, \infty)$, then $f(x) \cos \alpha x$ is an odd function and $f(x) \sin \alpha x$ is an even function in $(-\infty, \infty)$. consequently , the first integral on the right hand side of expression (*) vanishes and the expression becomes

$$\begin{aligned} F(\alpha) &= \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \sin \alpha x \, dx \\ &= \frac{2i}{\sqrt{2\pi}} \int_0^{\infty} f(x) \sin \alpha x \, dx \\ &= i \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin \alpha x \, dx \end{aligned}$$

Thus , for an odd function $f(x)$ defined in $(-\infty, \infty)$, we have

$$F(\alpha) = i F_s(\alpha)$$

Definition of Fourier cosine Transforms :

For a given $f(x)$, defined for $x>0$, Fourier cosine Transform is denoted by $F_c(\alpha)$ and defined as

$$F_c(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx$$

If the Fourier cosine Transform $F_c(\alpha)$ of $f(x)$ is known , then $f(x)$ is called the inverse Fourier cosine Transform of $F_c(\alpha)$ and defined as $f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_c(\alpha) \cos \alpha x \, d\alpha$.

Relationship between $F(\alpha)$ and $F_c(\alpha)$:

For a function $f(x)$ defined in $(-\infty, \infty)$, the Fourier transform is

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} \, dx = \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{\infty} f(x) \cos \alpha x \, dx + i \int_{-\infty}^{\infty} f(x) \sin \alpha x \, dx \right\} \dots \dots \dots (#)$$

Suppose $f(x)$ is an even function in $(-\infty, \infty)$, then $f(x) \cos \alpha x$ is an even function and $f(x) \sin \alpha x$ is an odd function in $(-\infty, \infty)$. consequently , the second integral on the right hand side of expression (#) vanishes and the expression becomes

$$\begin{aligned} F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cos \alpha x \, dx \\ &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx \end{aligned}$$

Thus , for an even function $f(x)$ defined in $(-\infty, \infty)$, we have

$$F(\alpha) = F_c(\alpha)$$

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V. APPLICATION TO INITIAL BOUNDARY VALUE PROBLEM (IBVP)

The solution of a IBVP consisting of a partial differential equation together with boundary and initial conditions can be solved by the Fourier Transform method . If the boundary conditions are of Dirichlet type where the function value is prescribed on the boundary , then the Fourier sine transform is used . If the boundary conditions are of Neumann type where the derivative of function is prescribed on boundary , then Fourier cosine Transform is used . In either case , the Partial Differential Equations reduces to an Ordinary Differential Equations in Fourier Transform which is solved . Then Fourier cosine or sine Transform will give the solution to the problem .

The Fourier Transform is also used in nuclear magnetic resonance (NMR) and in other kinds of spectroscopy ,example infrared (FTIR). In NMR an exponentially shaped free induction decay (FID) signal is acquired in the time domain and Fourier –transformed to a Lorentzian line-shape in the frequency domain .The Fourier Transform is also used in magnetic resonance imaging (MRI) and mass spectrometry .

Fourier Transform , which was first proposed to solve Partial Differential Equations(PDEs) such as Laplace , Heat and Wave equations has enormous applications in physics , engineering and chemistry . Some applications of Fourier Transform include

- 1) Communication : Fourier Transform is essential to understand how a signal behaves when it passes through filters , amplifiers and communication channels.
- 2) Image processing :Transformation , representation , encoding , smmoothing and sharpening images .Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components .comparing with the signal process , which is often using 1-dimensional Fourier Transform, in imaging analysis , 2 or higher dimensional Fourier Transform are being used .Fourier Transform has been widely applied to the fields of image analysis .
- 3) Data analysis :Fourier transform can be used as high – pass , low-pass , and band-pass filters and it can also be applied to signal and noise estimation by encoding the time series .
- 4) Cell phones : Every mobile device –netbook , notebook , tablet , and phone have been built in high speed cellular data connection , just like Fourier Transform . sine and cosine are keys to the success of Fourier Transform because sound may be represented by a complex combination of their waves. Humans very easily perform Fourier transform mechanically every day . For example , when you are in room with a great deal of noise and you selectively hear your name above the noise , you have just performed Fourier transform .

VI. CONCLUSION

This paper presents a brief overview of Fourier Transform. The primary use of Fourier Transform of converting a time domain function into its frequency domain . Major properties of Fourier Transform were discussed in detail. It also included a explanation of Inverse Fourier Transform..Fourier Transform is an essential component of much of modern applied mathematics. It forms an exceptionally powerful analytical tool for solving a broad range of partial differential equations. Fourier Transform has tremendous uses in many branches of Applied Sciences , physics and electrical power engineering

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